

WORDS TO KNOW

right triangle

hypotenuse

legs

Pythagorean theorem

converse of the
Pythagorean theorem

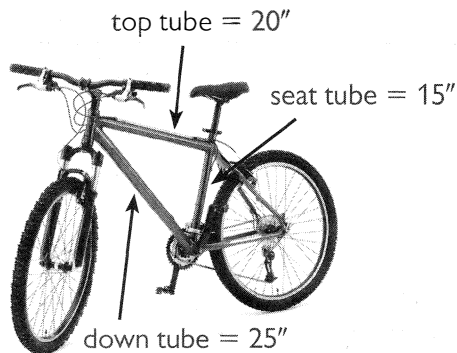
Lesson 28

UNDERSTAND THE PYTHAGOREAN THEOREM NY-8.G.6

INTRODUCTION

Real-World Connection

Lena is a bike mechanic. She knows that the top tube, down tube, and seat tube of many bike frames form a triangle. One day, she works on a bike with a frame that appears to form a right triangle. Lena knows the tube lengths, but she would like to know for sure if they form a right triangle. Let's practice the skills in the **Guided Instruction** and **Independent Practice** and, at the end of the lesson, find the answer to Lena's problem!



What I Am Going to Learn

- How to apply the Pythagorean theorem

What I May Already Know

- I can draw geometric shapes with given directions by freehand or with a protractor.

Vocabulary in Action

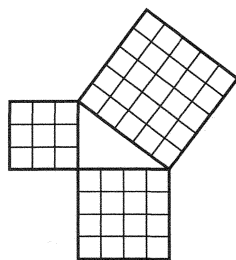
- A **right triangle** is a triangle with one right angle. The longest side, the side opposite the right angle, is the **hypotenuse**, and the two sides that form the right angle are called **legs**.
- The **Pythagorean theorem** states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. You can use an understanding of the Pythagorean theorem to solve problems.

We often see the Pythagorean theorem illustrated as below, where each side is squared, and the sum of the area of the smaller squares is equal to the area of the largest square:

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = 5^2$$

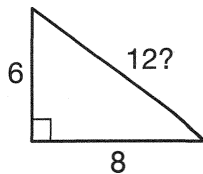
$$9 + 16 = 25$$



- The **converse of the Pythagorean theorem** states: If the sum of the squares of two sides of a triangle is equal to the square of the third side, the triangle is a right triangle.

EXAMPLE

Do the side lengths 6, 8, and 12 form a right triangle?



$$\text{Is } 6^2 + 8^2 = 12^2?$$

$$36 + 64 = 144?$$

$$100 \neq 144$$

There are a few conclusions:

- 1) The side lengths of 6, 8, and 12 units *do not* form a right triangle.
- 2) If a right triangle had legs of 6 and 8 units long, the hypotenuse would not be 12 units long.
- 3) Notice that $36 + 64 = 100$, and $100 = 10^2$. So, a right triangle with legs of 6 and 8 units long would have a hypotenuse of 10 units long.

THINK ABOUT IT

Use the converse of the Pythagorean theorem. If the equation is true, the side lengths form a right triangle.

GUIDED INSTRUCTION

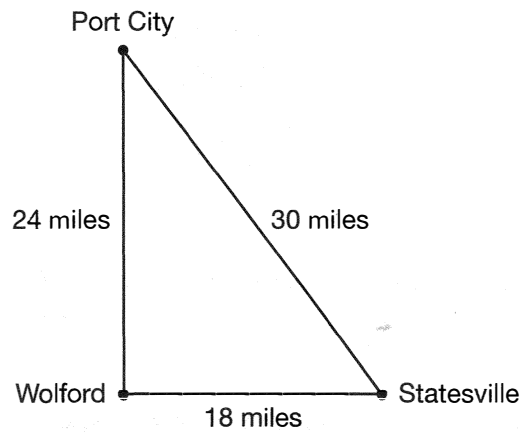
SKETCH IT

Start by drawing a right triangle that goes with the problem.

- Jonah sees three towns on a map that appear to form a right triangle. He checks the distances between each town. He finds that the distance to Statesville from Wolford is 18 miles, the distance from Wolford to Port City is 24 miles, and the distance from Port City to Statesville is 30 miles. Draw a diagram that shows the positions relative to one another. Do the three towns form a right triangle?

Step One Draw a triangle and label the measures given in the problem. Label the vertices of the triangle with the names of the towns.

The two shorter sides of the triangle are legs. The longer side is the hypotenuse.



Step Two Substitute the legs and hypotenuse lengths into the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

where a and b are the lengths of the legs and c is the length of the hypotenuse.

$$a = 18$$

$$b = 24$$

$$c = 30$$

$$18^2 + \boxed{} = \boxed{}?$$

Step Three Simplify the equation.

$$18^2 + 24^2 = 30^2?$$

$$324 + 576 = 900?$$

$$\boxed{} = 900 \checkmark$$

Step Four Solve the problem.

The sum of the squares of the legs is equal to the square of the hypotenuse. The towns do form a right angle.

2. A painter needs to use a 13-foot ladder to paint the side of a house. He places the ladder a distance of 5 feet from the side of the house. How far up the wall will the ladder reach?

Step One First draw a diagram of the situation described.

Step Two Determine what information is given and what you want to know. The diagram shows that the side of the house, ground, and ladder form a right triangle. You know the length of the ladder (hypotenuse) and the length of one leg of the triangle. You want to find the height of the wall where the ladder touches, which is the length of the other leg of the triangle.

Step Three Use the Pythagorean theorem to write an equation.

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = (\boxed{})^2$$

Solve for b .

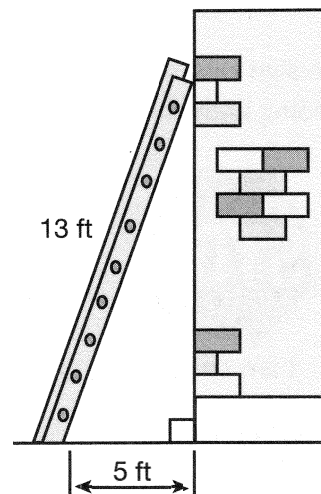
$$25 + b^2 = \boxed{}$$

$$b^2 = \boxed{} - 25$$

$$b^2 = \boxed{}$$

$$b = \sqrt{\boxed{}}$$

$$b = \boxed{}$$



Step Four Solve the problem.

The unknown leg length (b) is 12. The ladder will reach 12 feet up the wall.

3. Which set of side lengths describes a right triangle?

- (A) 2, 3, and 9
- (B) 6, 8, and 10
- (C) 7, 24, and 26
- (D) 10, 12, and 16

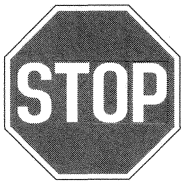
TIPS AND HINTS

Find the squares of the two shorter sides and add them together. Do they equal the square of the longer side?

Learning Together

Work in small groups to test the converse of the Pythagorean theorem. Determine the heights of the group members' different pens and pencils. Test the heights in sets of three to determine whether there is any combination that could result in a right triangle. Record your observations when the three heights are very close and when one is very different. Share your findings with the class.

Circle the sign that shows how you are doing with the skill.



I am stuck.



I almost have it.



I understand the skill.



How Am I Doing?

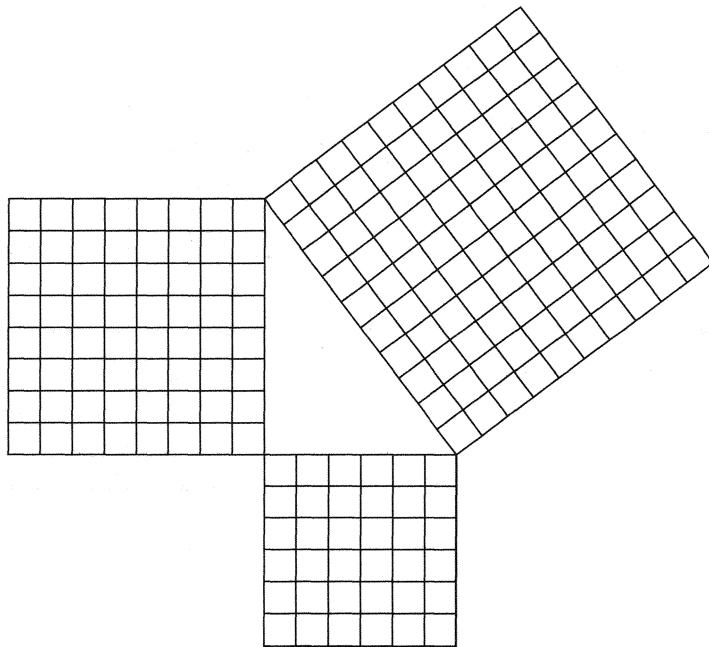
What questions do you have?

Where in the real world could you use the Pythagorean theorem?

What sides of the triangle do you add when using the Pythagorean theorem?

INDEPENDENT PRACTICE 1

- 1 The image below shows a visual model of the Pythagorean theorem.



◀ THINK ABOUT IT

Does the model represent both the leg lengths of a triangle and the Pythagorean theorem?

Sophie is using this model to find the hypotenuse of a triangle. How many units long is the hypotenuse?

- A 6
- B 8
- C 10
- D 14

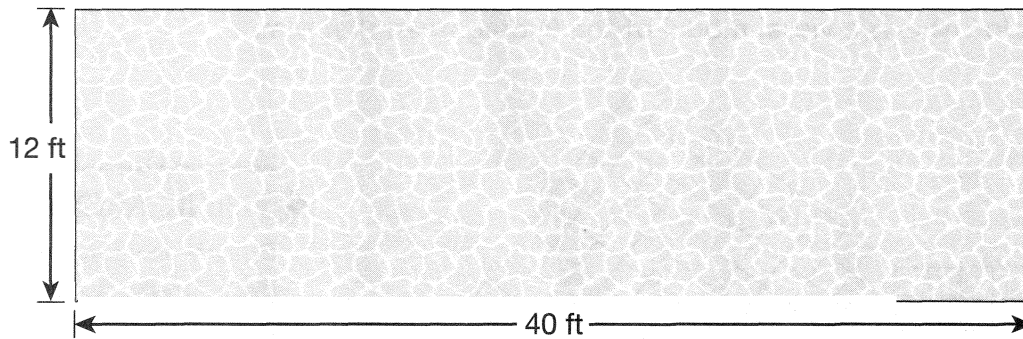
- 2 Which set of lengths could represent three sides of a right triangle?

- A 5, 5, 5
- B 3, 4, 6
- C 8, 11, 13
- D 9, 12, 15

◀ THINK ABOUT IT

Does the hypotenuse always have to be the longest of the three legs in a triangle?

- 3 The image below shows the Gallants' new rectangular swimming pool, which has a length of 40 feet and a width of 12 feet.



Tamika swims the diagonal distance across the pool. What is the **best** estimate of the distance, in feet, she swims?

- A 38
B 41
C 42
D 52

◀ SKETCH IT

To help clarify the situation, draw on the pool the path Tamika swims.

- 4 Kimi is making a flag with two right triangles on it. The larger triangle will have a hypotenuse of 40 inches. The smaller triangle will have a hypotenuse of 30 inches. Each will have one leg that is 24 inches.

What are the other leg lengths of the triangles? Use the Pythagorean theorem to explain your answer.

Larger triangle _____

Smaller triangle _____

Explain your answer.

◀ TIPS AND HINTS

The Pythagorean theorem does not work with all triangles. It only works with right triangles.

INDEPENDENT PRACTICE 2

1 Triangle ABC is a right triangle. Which statement below must be true?

- A It has one right angle.
- B All three of its sides are different lengths.
- C The side opposite the right angle is a leg.
- D The sum of the square of the hypotenuse and the square of one leg is equal to the square of the other leg.

2 The lengths of the legs of a right triangle are 8 and 10 units. What is the length of the hypotenuse, in units, to the nearest whole unit?

- A 12
- B 13
- C 18
- D 80

3 Triangle MNO is a right triangle. The length of one leg is 4 and the length of the other leg is 7. What is the length of the hypotenuse?

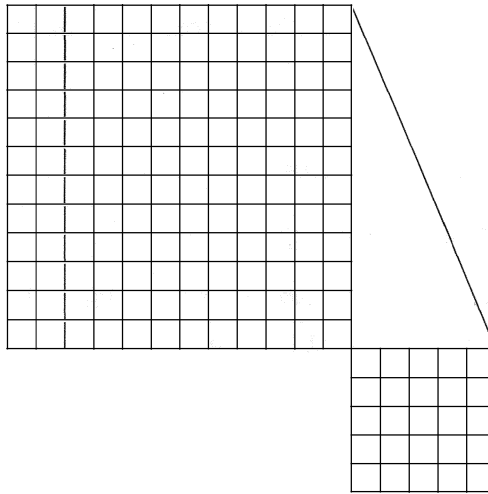
- A $\sqrt{11}$
- B $\sqrt{28}$
- C $\sqrt{56}$
- D $\sqrt{65}$

4 Which set of side lengths describes a right triangle?

- A 1, 3, and 4
- B 33, 44, and 55
- C 50, 75, and 80
- D 5, 12, and 23

5

Mateo made the drawing below. The small squares are all the same size.

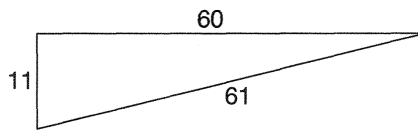


If he also aligns a large square along the hypotenuse, how many small squares will fit in the new square?

- A 13
- B 119
- C 169
- D 196

6

An engineer has designed the triangular machine part below.



Sets of positive whole numbers that work in the Pythagorean theorem are called Pythagorean triples. Did the engineer design a part with sides that are a Pythagorean triple?

- A Yes, because $11^2 + 60^2 = 61^2$.
- B Yes, because $11 + 60 > 61$, $11 + 61 > 60$, and $60 + 61 > 11$.
- C No, because the triangle is not notated as a right triangle.
- D No, because $11 + 60 > 61$.

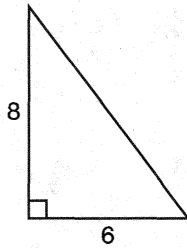
7

Which choice describes a right triangle?

- A a piece of steel with sides 2 m, 4 m, and 5 m
- B a stained glass window with sides 8 in., 15 in., and 20 in.
- C a garden with sides 12 ft, 34 ft, and 37 ft
- D a ceramic tile with sides 20 cm, 21 cm, and 29 cm

8

The right triangle below has legs 6 and 8 units long.



How can you use the Pythagorean theorem to find the hypotenuse?

Explain your answer.

What is the hypotenuse?

Answer _____

9

An isosceles triangle has side lengths of 7, 7, and 10. Use math to show that the triangle would **not** be a right triangle.

Show your work.

What could be done to the hypotenuse to make the triangle a right triangle?

Explain your answer.

EXIT TICKET

NY-8.G.6

Now that you have mastered applying the Pythagorean theorem, let's solve the problem in the Real-World Connection.

Lena is a bike mechanic. She knows that the top tube, down tube, and seat tube of many bike frames form a triangle. One day, she works on a bike with a frame that appears to form a right triangle. Lena knows the tube lengths, but she would like to know for sure if they form a right triangle. Use the information in the diagram to determine if the tubes form a right angle.

